XXVth Congress and Plenary Assembly of Split
(22-30 October 1976)

Physical Oceanography Committee

ON NEGLECTING OF ACCELERATION ADVECTIVE MEMBER IN THE HYDRODYNAMICAL EQUATIONS

Mario A. BONE
Institute of Oceanography and Fisheries, Split, Yugoslavia

ABSTRACT

It is demonstrated that the effects from the short wave part of the flow field spectrum generated by the quadratic member of momentum advection in Euler hydrodynamical equations in first approximation (Eckart's meaning) are negligible.

RESUME

On démontre qu'en première approximation (selon Eckart), les effets de la partie du spectre des ondes courtes, engendrés par le terme carré de l'advection de quantité de mouvement dans les équations hydrodynamiques d'Euler, sont négligeables.

Assume that (1) $\vec{v}(\vec{r},t) = \sum_{\vec{k},\omega} \vec{V}(\vec{k},\omega) \exp(i\vec{k}.\vec{r} + i\omega t)$
where $\omega = \frac{2\pi m_t}{T}$, $\vec{k} = 2\pi\left(\frac{m_x}{L_x}, \frac{m_y}{L_y}, \frac{m_z}{L_z}\right)$ $(m_t, m_x, m_y, m_z = 0, \pm 1, \pm 2 \ldots)$
and T is time interval wherein the flow field is observed in the space domain $x \in [x_o, x_o+L_x], \ldots\ldots$ and $z \in [z_o, z_o+L_z]$
$\vec{V}(\vec{k},\omega)$ is spatial-temporal spectrum of $\vec{v}(\vec{r},t)$ in the domain observed. If $\vec{f}(\vec{r},t)$ is the force that affects the unit mass of fluid with gravity center at $\vec{r}$ and time t, it will be (2) $\frac{\partial \vec{v}}{\partial t} + \vec{v}.\nabla\vec{v} = \vec{f}$. On the basis of (1),(2)

may be rewriten as $i\Omega\,\vec{V}(\vec{K},\Omega)+\sum_{\vec{k},\omega}\vec{V}(\vec{K}-\vec{k},\Omega-\omega)\cdot i\vec{k}\,\vec{V}(\vec{k},\omega)$ $=\vec{F}(\vec{K},\Omega)$ (3) where the equation (3) and its sum have been obtained as a form of convolution theorem in the folowing manner : $\vec{V}.\nabla\vec{V}=$

$$=\sum_{\vec{k},\omega}\sum_{\vec{k}',\omega'}\vec{V}(\vec{k}',\omega')\cdot i\vec{k}\,\vec{V}(\vec{k},\omega)\exp\left[i(\vec{k}'+\vec{k}).\vec{r}+i(\omega+\omega')t\right] \qquad (4).$$

By introduction of $\vec{K}=\vec{k}+\vec{k}'$, $\Omega=\omega+\omega'$ the sought form is obtained for $\vec{K}$ and Ω by which the equation has been decomposed, taking account of linear independence of exponential functions. In (2) the force may be averaged by a spatial area $V=\ell_x\ell_y\ell_z$ where $x\in\left[x'-\frac{\ell_x}{2},x'+\frac{\ell_x}{2}\right],\ldots\ldots$ and $z\in\left[z'-\frac{\ell_z}{2},z'+\frac{\ell_z}{2}\right]$ is domain by which the force is averaged so that we have $\vec{f}_1(\vec{r}',t)=\frac{1}{V}\int\vec{f}\,dV$ (5) and $\vec{f}=\vec{f}_1+\vec{f}_2$ (6).

The spatial distribution spectrum $\vec{f}_1$ will rapidly vanish for all the wave lenghts lower than ℓ_x,ℓ_y,ℓ_z and for higher wave lenghts it will remain very slightly changed. Spectrum $\vec{f}_1$ is complementary to the spectrum $\vec{f}_2$ in relation to $\vec{f}$ spectrum. It is already known that solutions that satisfy the equation (7) $\frac{\partial\vec{V}_1}{\partial t}=\vec{f}_1$ are good approximative solutions for the flow field if $\vec{f}_2=0$ and ℓ_x,ℓ_y,ℓ_z are high enough in relation to flow velocity and if in the initial conditions there is not component of flow field from shortwave part of the spectrum. Solution $\vec{V}_1$ will rapidly converge to zero in the shortwave part of the spectrum as the force $\vec{f}_1$. If we write the solution of (2) like (8) $\vec{V}=\vec{V}_1+\vec{V}_2$ the equation (2) for $\vec{f}_2\neq 0$ will be (9) $\frac{\partial\vec{V}_2}{\partial t}+\vec{V}.\nabla\vec{V}=\vec{f}_2$.

If we accept the condition that $\vec{V_2}$ is either of order of magnitude of $\vec{V_1}$ or lower, considering only those solutions of (2) whose velocity are small in relation to ℓ_x, ℓ_y, ℓ_z, and the member $\vec{V_2} \cdot \nabla \vec{V_1}$ may be rejected by the same order of approximation like done previously for the long-wave part of the spectrum $(\vec{k} \approx 0)$ we will obtain

(10) $$\left(\frac{\partial \vec{V_2}}{\partial t} + \vec{V_2} \cdot \nabla \vec{V_2} \right) = - \left(\vec{V_1} \cdot \nabla \vec{V_2} \right).$$

Like in the equation (3) it may be writen as (11)

$$\left(\vec{V_1} \cdot \nabla \vec{V_2} \right)_{\vec{k} \approx 0} = \sum_{\vec{k} \approx 0,\, \omega} \left\{ \sum_{\vec{K},\, \Omega} \vec{V_1}(\vec{k}-\vec{K}, \omega-\Omega) \cdot i \vec{K} \vec{V_2}(\vec{K}, \Omega) \right\} \exp(i\vec{k}\vec{r}+i\omega t)$$

and as $\vec{k} \approx 0$ is observed and $\vec{V_1}$ differs from zero only for $\vec{k} - \vec{K} \approx 0$ so by the same order of approximation it may be writen as follows (12) $\left(\frac{\partial \vec{V_2}}{\partial t} + \vec{V_2} \cdot \nabla \vec{V_2} \right)_{\vec{k} \approx 0} = 0$.

As by the equation (7) we have obtained the approximative solution in the longwave part of the spectrum of the flow field, irrespective of the field of non-observed forces and movements in the short wave part of the spectrum. Equation (12) indicates that the solution may differ from rigorous solution, within long wave approximation, for only one additive inertial flow field that may be analysed irrespective of the studied longwave part. Inertial field may be neglected by conveniently chosen boundary and initial conditions. It may be also observed as a stationary flow field along its stream line homogeneous or in other way suitable.