A THEORETICAL VIEW OF OCEANIC AND MEDITERRANEAN VORTICES

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The recent Mode experiments have raised a vivid interest in oceanic eddies[1]. A simple steady two-dimensional inviscid model is now described[2]. It is then used both to investigate stability properties and to describe Mediterranean systems.

A first look can show that many existing systems have the character of an eddy, but that they are not sphereically symmetric. Then one can introduce a stream function

$$\Psi(x,y) = A\left(\sqrt{x^2+y^2}\right) + B(y)$$

where (x=0, y=0) is point in the ocean and (x-y) are cartesian coordinates with y directed northward. By imposing that Ψ is a solution of the inviscid equation of motion for a rotating system, one obtaines in β or β-β plane approximation

$$\Psi = A J_0\left(K\sqrt{x^2+y^2}\right) + B\cos Ky + C\sin Ky - \frac{f(y)}{K^2} + D$$

where A,B,C,D are constants, J_0 is the Bessel function and f(y) is the Coriolis force.

This gives an oceanic eddy superimposed to a current, essentially to balance the deforming effect of the earth's rotation.

This model is rather simple but it allows enough freedom to study secondary effects[3]. One can easily see that many of the detected oceanic eddies have a rather irregular

pattern [1], probably not stationary in time. To see this effect, linearized time dependent stability theory can be studied. This implies

$$\Psi(x,y) \rightarrow A(\sqrt{x^2+y^2}) + B(y) + e^{\omega t} d(x,y) + e^{\omega^* t} d^*(x,y)$$

then it is possible to show on mathematical grounds that, for small d(x,v), the system can oscillate around the steady solution $= A + B$, but that these oscillations (if confined inside the eddy) remain bounded[3]. A stronger result[4] holds if one assumes f = constant, as a nonlinear theory of ARnold can be applied. It gives a general criterion of stability à la Liapunov, that for smaller oceanic eddies (10 km of radius) guarantees the barotropic stability.

About the Mediterranean area, eddy shapes in the density field in the Gulf of Lion are well known[5]. A possible other eddy shape, always in the density field, has been detected by Di Maio and Trotti, but later investigations have failed to recognize it[6]. A different pattern has been seen by Garzoli and Maillard between Sardinia, Sicily and the African coast. These systems are presumably stable and in some sense govern the general circulation of the Mediterranean Sea.

A smaller eddy shape (10 km of radius) has been recently studied by Gascard and Jeanin[5]. Theoretically, this is particularly interesting because of its remarkable stability, guaranteed by Arnold's theory. This effect being barotropic, one can also show that the baroclinic stability is much more doubtful for the most external part. This kind of systems can then show a different, but not less important, kind of weakness.

References

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