

## A Simple Mathematical Model for Sediment Transport

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Sediment transport in water is mostly studied by using purely empirical methods. But in order to predict sedimentary deposit or erosional effects, one would like to have some theoretical tools for the description of sediment motion.

We consider a two-dimensional  $(x, z)$  stationary model, where the main sediment flow is horizontally ( $x$ -axis) and where the ground has a distance  $z = h(x)$  from the surface.

Using diffusion theory one derives - with some physically justified simplifications - from the continuity equation (sedimentary mass conservation) the following parabolic partial differential equation for the mean sediment concentration in suspension  $c(x, z)$

$$u \frac{\partial c}{\partial x} + (w - w_s) \frac{\partial c}{\partial z} - \frac{\partial}{\partial z} (\epsilon_s \frac{\partial c}{\partial z}) = 0 \quad (1)$$

with some appropriate initial and boundary conditions.

$u$  and  $w$  are the  $x$ - resp.  $z$ -components of the velocity field of the current. The coefficient  $\epsilon_s$  takes into account the sedimentary exchange due to turbulent flux. Using a lot of experimental data KERSSSENS [2] gave a suitable formula for  $\epsilon_s(z)$ .

The constant  $w_s$  characterizes the mean sedimentation speed, which is a function of the diameter, form and mass of the sediment particles and of the Reynolds number.

The mean velocity field  $(u, w)$  could of course be determined by solution of the associated time-independent Navier-Stokes equations for a viscous incompressible fluid. Since they are valid only for a laminary current, one could superpose a turbulent motion in adding a term taking into consideration the Reynolds tensions and the laminary layer phenomena.

Instead of choosing this complicated approach necessitating the resolution of a nonlinear system of partial differential equations, we follow an idea of PRANDTL [3] for the description of the shear-stresses. Introducing a roughness function for the description of the ground structure, as it has been done by J. NIKURADSE (see [4]), one can find by elementary integrations that

$$u = \frac{K \ln(z/z_0)}{\ln(h/z_0) - 1} \quad (2)$$

and

$$w = \frac{K \ln(h/z_0) \frac{dh}{dx}}{h \ln(h/z_0) - 1} (z \ln(z/z_0) - z + z_0), \quad (3)$$

where the constants  $K$  and  $z_0$  depend on  $h(x)$  and the roughness of the ground.

If one uses (2) and (3), the diffusion equation (1) can be solved numerically by standard discretisation techniques.

Knowing the local sedimentary density  $c(x, z)$  and the current field  $(u, w)$ , one easily calculates the total transport of the suspended particles.

Beyond that the motion of drift materials can be treated by the Engelund-Hansen method [1], which is also applied for finding the necessary boundary conditions at the ground level.

## REFERENCES.

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## Interstitial Water of Tyrrhenian Sea, Western Mediterranean

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This study aimed to demonstrate the characteristics and type of interstitial water, as well as the paleoenvironment and diagenetic processes governing the western Mediterranean region during the Holocene time. Seven core samples have been collected from Tyrrhenian Sea (Fig.1) using a stainless steel gravity core sampler of 4 meter length and 65mm diameter. Titanium hydraulic squeezers with pressure up to 200Kg/Cm2 have been used to extract the interstitial water from the sediments (Kriukov and Manheim, 1982). The interstitial water was analysed for salinity, alkalinity, SO<sub>4</sub>, Ca, Mg, Na and K. Measurements of Redox potential



(Fig.1) Location Map for core samples in Tyrrhenian Sea (V.Vavilov volcano, M.Marsili volcano, S.Stromboli volcano).

for some samples revealed that the sediments under investigation have been exposed to diagenesis due to aerobic conditions. Such diagenesis generally leads to very limited changes - or almost none at all - in the interstitial water, where it retains the original composition as sea water. According to Valyashko (1955), the interstitial water of Tyrrhenian Sea could be classified as oceanic type (MgSO<sub>4</sub>). Similar conclusion has been reached by the authors in 1988 concerning Nile Cone sediments, Southern Mediterranean. Normal values of salinity were found in the investigated basin, except in the southern part where higher values were recorded (i.e. up to 44.33 ‰). In addition, higher values of SO<sub>4</sub>, Na and K were observed in this part of Tyrrhenian. Stromboli volcano, which is active until now may play a dominant role in this respect. Infiltration of brines from the underlying Messinian evaporites have to be in consideration too. Alkalinity showed a slight decrease with depth in sediment successions in the northern part of the basin (cores No. 71, 72, 73 and 74), on the other hand, increased in the southern part. Generally, the low values of alkalinity observed in the interstitial water of the Tyrrhenian Sea could be attributed to the following reasons: 1-The precipitation of HCO<sub>3</sub> and CO<sub>3</sub> from the interstitial water as CaCO<sub>3</sub> minerals, i.e. aragonite and calcite. 2-Absence of sulphate reduction which prevents the accumulation of HCO<sub>3</sub> in interstitial water (SO<sub>4</sub>+2C+2H<sub>2</sub>O → 2HCO<sub>3</sub>+H<sub>2</sub>S). This phenomenon is due to the low content of organic matter which is the case in the investigated sediments. 3-Leaching of gypsum (CaSO<sub>4</sub>) from biogenic carbonate sediments. This gypsum decreases the solubility of CaCO<sub>3</sub> and consequently, ceases the accumulation of HCO<sub>3</sub> in the interstitial water (Shishkina, 1972).

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