Theoretical determination of the fractal dimension of fluid parcel trajectories in large and meso-scale flows

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Recent work suggests that Lagrangian parcel trajectories in large and meso-scale flows can have a fractal dimension (OSBORNE, et al, 1986, 1989; PROVENZALE, et al, 1989). Drifter trajectories may be viewed as fractal curves in the plane of the ocean's surface and have typical fractal dimensions of about 1.30 ± 0.06 in a range of scales normally attributed to geostrophic turbulence. There are several fundamental questions which we address with regard to these results (OSBORNE and PROVENZALE, 1989; OSBORNE and CAPONIO, 1990a, b):

(1) One normally thinks of solutions to partial differential equations as being reasonably smooth, well-behaved differentiable functions. How can it theoretically be possible for the *dynamical* motions of particle trajectories to *physically* be fractal curves?

(2) What are the *physical implications* of fractal trajectories on fluid flows in general?

(3) What new, *unique physical information* can the fractal dimension give us with regard to large scale flows?

(4) Can the fractal dimension be of use in solving the *inverse problem*, i.e. determination of the general circulation from drifter trajectories?

(5) What relation does the fractal dimension have to fluid properties such as *anisotropic and anomalous diffusion* in fluids?

(6) What are the implications on *fractal front propagation* and on *fractal frontogenesis*?

To address these questions we discuss a simple, nonlinear Hamiltonian model for describing particle motions in large and meso-scale oceanic and atmospheric flows. The model predicts nonlinear fluid parcel trajectories in 2-D where the stream function power spectrum has the form $k^{-\gamma}$ (γ const), while the velocity spectrum is given by $k^{-\delta}$, $\delta=\gamma-3$. The equations of motion are

$$\dot{\mathbf{x}} = u_{rms}\hat{\mathbf{k}}\times\nabla\psi(\mathbf{x},t)$$

The over-dot denotes time derivative, $\mathbf{x}(t)=[x(t),y(t)]$ is the parcel position in the xy-plane, $\hat{\mathbf{k}}$ is the unit vector perpendicular to this plane, $\nabla = (\partial_x,\partial_y,\partial_z)$ and u_{rms} is the assumed rms velocity of the flow. $\psi(\mathbf{x},t)$ is the stream function or Hamiltonian, which is here given a stochastic representation in two dimensions. The dispersion relation is $\omega = uk$, where $k = |\mathbf{k}|$, and u is constant. Using this model as a basis we apply both analytical and numerical methods to show that fluid particle trajectories: (1) while differentiable and chaotic at small scale, may be *fractal space curves* at larger scale and (2) undergo *anomalous transport*.

We identify a "stochasticity parameter" $\mu=u/u_{rms}$ which characterizes the flow: (a) When $\mu = 0$ the Hamiltonian is time independent and exactly integrable, but the system is nevertheless fully nonlinear. For specific initial conditions $\mathbf{x}(0)=\mathbf{x}_0$, $\mathbf{x}(t)$ lies on the contour given by $\psi(\mathbf{x}_0)$=const. For $0\le\delta\le1$, $\mathbf{x}(t)$ is a fractal curve in the xy-plane with dimension $D=2/(\delta+1)$, $1\le D\le2$, and has *anomalous absolute diffusion* $\langle|\mathbf{x}(t) - \mathbf{x}(0)|^2\rangle = \mathfrak{D}t^{2/D}$, for $\mathfrak{D}$ the diffusion coefficient. (b) For $\mu << 1$ and finite, the flow is chaotic and of *KAM* (Kolmogorov-Arnol'd-Moser) *type*, i.e. the Hamiltonian consists of $\psi(\mathbf{x})$ plus a perturbation. The KAM "surfaces" are very nearly fractal contours of $\psi(\mathbf{x})$. $\psi(\mathbf{x},t)$ varies slowly in time and stochastic layers are formed. (c) For $\mu < 1$, the flow is chaotic and lies in a *trapping regime* characterized by *vortices* and *vortex hopping*. (d) When $\mu \lesssim 1$, clustering occurs, and isolated clumps of activity appear, evidently the remnants of the vortices. (e) For $\mu \gtrsim 1$, the flow is *fully stochastic*, equivalent to a *nongaussian random walk* in the plane. Example trajectories are given in Figure 1.

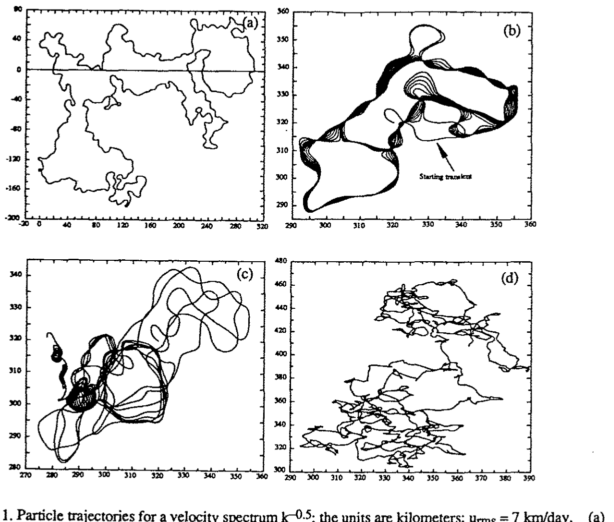

Figure 1. Particle trajectories for a velocity spectrum $k^{-0.5}$; the units are kilometers; u_{rms} = 7 km/day. (a) μ=0, the motion is periodic, and the trajectory is a fractal curve with dimension D=1.33. (b) μ=0.0014, the motion is of KAM type and stochastic layers form. (c) μ=0.014, the motion lies in the vortex/vortex-hopping regime. (c) μ=1.4, the motion is fully stochastic, a nongaussian random walk in the plane.

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Experimental Study of nonlinear internal waves in infinite or semi-infinite ocean

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Experiments were performed on the large 14m diameter rotating platform, and showed that an important parameter for long nonlinear internal waves is the ratio of a characteristic length of the wave upon the internal Rossby radius of deformation. These experiments suggested new theoretical developments, in order to get an unified view of both linear and nonlinear waves in rotating fluid.

From the experiments, it appeared clearly that, in infinite rotating fluid, when the rotation is strong, i.e. when the Rossby radius of deformation is smaller than, or of the same order of magnitude as the characteristic wave-length, there is no solitary waves, but only dispersive waves, and the analysis shows that there is also the possibility of periodic waves, propagating faster than the critical phase-speed, and with a horizontal crest, i.e. Sverdrup waves. In infinite fluid, when the rotation is weak or very weak, i.e. when the Rossby radius of deformation is larger or much larger than the characteristic length, then there is either solitary waves solutions of the Ostrovskiy equation, or solitary waves solutions of the Korteveg-de Vries equation, respectively. Experimentally, it is easy to show that there exist solitary waves, with an horizontal crest, propagating with a celerity which is function of the amplitude, and a characteristic length inversely proportional to the square root of the amplitude, i.e. fulfilling the K.d.V. conditions. But with the wave generator that we used, we could not observe Ostrovskiy solitary waves.

In semi-infinite ocean, for all cases, we observed nonlinear Kelvin waves, propagating along the side-wall. But the shape of the wave greatly depends of the initial condition. When this condition is bi-dimensional, as in the infinite ocean, then the wave crest is curved backward, as in a channel, and that curvature is likely due to the superposition of a Kelvin solitary wave and Poincaré waves propagating at the same phase-speed. Actually, it can be shown that Poincaré waves are but superpositions of Sverdrup waves propagating in two symetrical directions. But when the initial condition is three-dimensional, and roughly correspond to a Kelvin wave, then what is observed downstream is a nonlinear Kelvin wave, with a crest perpendicular to the side, and propagating with a celerity faster than the critical phase-speed.

To our knowledge, it is the first time that such a complete set of experimental data and analytical developments is available, for long nonlinear waves in rotating fluid.

Now, if the generation mechanism is a cylindrical body moving along a vertical wall and all located in the lower layer, if the lower layer is thinner than the upper layer, then the body generates nonlinear Kelvin waves which propagate downstream in front of the body, as in a channel, and their crest is curved backward. The amplitude and number of waves observed at a given place downstream the body, is function of the distance between the body and the wall, as well as of the diameter of the body. If the lower layer is thicker than the upper layer, then there is no wave generation, but only an upwelling generation, and the front of this upwelling moves at the critical phase speed. This models the upwelling and waves generated by an island located near a coast. Some experiments were also realized with a cape, and gave similar results.