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A Simple Mathematical Model for Sediment Transport

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Sediment transport in water is mostly studied by using purely empirical methods. But in order to predict sedimentary deposit or erosional effects, one would like to have some theoretical tools for the description of sediment motion.

We consider a two-dimensional (x, z) stationary model, where the main sediment flow is horizontally (x-axis) and where the ground has a distance $z = h(x)$ from the surface.
Using diffusion theory one derives - with some physically justified simplifications - from the continuity equation (sedimentary mass conservation) the following parabolic partial differential equation for the mean sediment concentration in suspension $c(x, z)$

$$u\frac{\partial c}{\partial x} + (w - w_s)\frac{\partial c}{\partial z} - \frac{\partial}{\partial z}(\epsilon_s \frac{\partial c}{\partial z}) = 0 \qquad (1)$$

with some appropriate initial and boundary conditions.
u and w are the x- resp. z-components of the velocity field of the current. The coefficient ϵ_s takes into account the sedimentary exchange due to *turbulent* flux. Using a lot of experimental data KERSSENS [2] gave a suitable formula for $\epsilon_s(z)$.
The constant w_s characterizes the mean sedimentation speed, which is a function of the diameter, form and mass of the sediment particles and of the Reynolds number.

The mean velocity field (u, w) could of course be determined by solution of the associated time-independent Navier-Stokes equations for a viscous incompressible fluid. Since they are valid only for a laminary current, one could superpose a turbulent motion in adding a term taking into consideration the Reynolds tensions and the laminary layer phenomena.
Instead of choosing this complicated approach necessitating the resolution of a nonlinear system of partial differential equations, we follow an idea of PRANDTL [3] for the description of the shear-stresses. Introducing a roughness function for the description of the ground structure, as it has been done by J. NIKURADSE (see [4]), one can find by elementary integrations that

$$u = \frac{K \ln(z/z_0)}{\ln(h/z_0) - 1} \qquad (2)$$

and

$$w = \frac{K \ln(h/z_0)\frac{dh}{dx}}{h \ln(h/z_0) - 1)}(z \ln(z/z_0) - z + z_0), \qquad (3)$$

where the constants K and z_0 depend on $h(x)$ and the roughness of the ground.

If one uses (2) and (3), the diffusion equation (1) can be solved numerically by standard discretisation techniques.

Knowing the local sedimentary density $c(x, z)$ and the current field (u, w), one easily calculates the total transport of the suspended particles.

Beyond that the motion of drift materials can be treated by the Engelund-Hansen method [1], which is also applied for finding the necessary boundary conditions at the ground level.

REFERENCES.

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[4] SCHLICHTING, H., "Grenzschicht-Theorie", Karlsruhe 1965