Theoretical determination of the fractal dimension of fluid parcel trajectories in large and meso-scale flows

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Recent work suggests that Lagrangian parcel trajectories in large and meso-scale flows can have a fractal dimension (OSBORNE, et al, 1986, 1989; PROVENZALE, et al, 1989). Drifter trajectories may be viewed as fractal curves in the plane of the ocean's surface and have typical fractal dimensions of about 1.30 ± 0.06 in a range of scales normally attributed to geostrophic turbulence. There are several fundamental questions which we address with regard to these results (OSBORNE and PROVENZALE, 1989; OSBORNE and CAPONIO, 1990a, b):

(1) One normally thinks of solutions to partial differential equations as being reasonably smooth, well-behaved differentiable functions. How can it theoretically be possible for the *dynamical* motions of particle trajectories to *physically* be fractal curves?

(2) What are the *physical implications* of fractal trajectories on fluid flows in general?

(3) What new, *unique physical information* can the fractal dimension give us with regard to large scale flows?

(4) Can the fractal dimension be of use in solving the *inverse problem*, i.e. determination of the general circulation from drifter trajectories?

(5) What relation does the fractal dimension have to fluid properties such as *anisotropic and anomalous diffusion* in fluids?

(6) What are the implications on *fractal front propagation* and on *fractal frontogenesis*?

To address these questions we discuss a simple, nonlinear Hamiltonian model for describing particle motions in large and meso-scale oceanic and atmospheric flows. The model predicts nonlinear fluid parcel trajectories in 2-D where the stream function power spectrum has the form $k^{-\gamma}$ (γ const), while the velocity spectrum is given by $k^{-\delta}$, $\delta=\gamma-3$. The equations of motion are

$$\dot{\mathbf{x}} = u_{rms}\hat{\mathbf{k}}\times\nabla\psi(\mathbf{x},t)$$

The over-dot denotes time derivative, $\mathbf{x}(t)=[x(t),y(t)]$ is the parcel position in the xy-plane, $\hat{\mathbf{k}}$ is the unit vector perpendicular to this plane, $\nabla = (\partial_x,\partial_y,\partial_z)$ and u_{rms} is the assumed rms velocity of the flow. $\psi(\mathbf{x},t)$ is the stream function or Hamiltonian, which is here given a stochastic representation in two dimensions. The dispersion relation is $\omega = uk$, where $k = |\mathbf{k}|$, and u is constant. Using this model as a basis we apply both analytical and numerical methods to show that fluid particle trajectories: (1) while differentiable and chaotic at small scale, may be *fractal space curves* at larger scale and (2) undergo *anomalous transport*.

We identify a "stochasticity parameter" $\mu=u/u_{rms}$ which characterizes the flow: (a) When $\mu = 0$ the Hamiltonian is time independent and exactly integrable, but the system is nevertheless fully nonlinear. For specific initial conditions $\mathbf{x}(0)=\mathbf{x}_o$, $\mathbf{x}(t)$ lies on the contour given by $\psi(\mathbf{x}_o)$=const. For $0\leq\delta\leq1$, $\mathbf{x}(t)$ is a fractal curve in the xy-plane with dimension $D=2/(\delta+1)$, $1\leq D\leq2$, and has *anomalous absolute diffusion* $\langle|\mathbf{x}(t)-\mathbf{x}(0)|^2\rangle = \mathfrak{D}t^{2/D}$, for $\mathfrak{D}$ the diffusion coefficient. (b) For $\mu \ll 1$ and finite, the flow is chaotic and of *KAM* (Kolmogorov-Arnol'd-Moser) *type*, i.e. the Hamiltonian consists of $\psi(\mathbf{x})$ plus a perturbation. The KAM "surfaces" are very nearly fractal contours of $\psi(\mathbf{x})$. $\psi(\mathbf{x},t)$ varies slowly in time and stochastic layers are formed. (c) For $\mu < 1$, the flow is chaotic and lies in a *trapping regime* characterized by *vortices* and *vortex hopping*. (d) When $\mu \lesssim 1$, clustering occurs, and isolated clumps of activity appear, evidently the remnants of the vortices. (e) For $\mu \gtrsim 1$, the flow is *fully stochastic*, equivalent to a *nongaussian random walk* in the plane. Example trajectories are given in Figure 1.

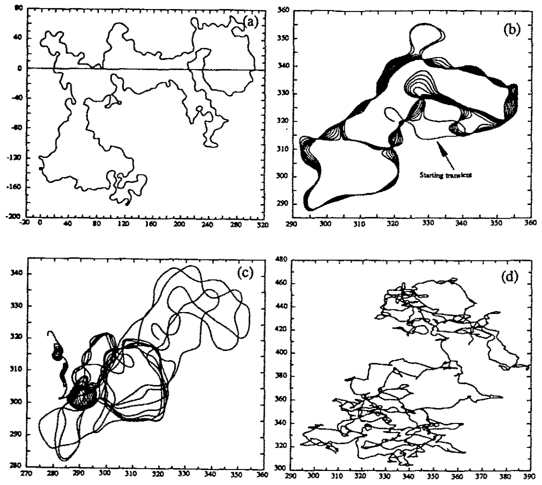

Figure 1. Particle trajectories for a velocity spectrum $k^{-0.5}$; the units are kilometers; u_{rms} = 7 km/day. (a) μ=0, the motion is periodic, and the trajectory is a fractal curve with dimension D=1.33. (b) μ=0.0014, the motion is of KAM type and stochastic layers form. (c) μ=0.014, the motion lies in the vortex/vortex-hopping regime. (c) μ=1.4, the motion is fully stochastic, a nongaussian random walk in the plane.

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