TURBULENT STRUCTURE FUNCTIONS IN GEOPHYSICAL FLOWS

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Abstract

In geophysical flows, there are many instances, where turbulence is originated locally, such as in surface wave breaking at the surfzone or by internal wave breaking in the lee of a mountain. The use of velocity structure functions and their moments may give an indication of the spatial and time delay from the source of turbulence. The variation of the structure functions and the scaling exponents in decaying non-homogeneous turbulence flows produced by a grid is investigated by means of sonic velocimeter SONTEK3-D. In the analysis we invoke the concept of the Extended Self Similarity (ESS) and find that there are changes in the structure functions related to the intermittency.

Keywords: Turbulence

Introduction

In recent years, new interest has emerged concerning the scaling properieties of turbulence flows. They are reflected in the scale invariance of Navier-Stokes equations, both in two dimensions (2D) and three dimensions (3D).The statistical behaviour of two-dimensional and three-dimensional fully developed turbulence at large and small scales has been investigated as shown by Sreenivasan [1]. A common way to approach this problem is through the velocity structure functions. Usually the scaling properties of moments of velocity differences at the scale r $S_p(r)$ are investigated using the following definition:

$$S_p(r) = \langle |V(x+r) - V(x)|^p \rangle = \langle |\delta V(r)|^p \rangle \quad (1).$$

where $\langle .... \rangle$ stands for ensemble average and the V is the velocity component parallel to r. At high Reynold number, $Re = U_o L/\nu$ the structure function $S_p(r)$ satisfies the relationship :

$$S_p(r) \propto r^{\zeta p} \quad (2).$$

For turbulence within the inertial subrange $L > r >> \eta$ where the L is the integral scale, $\eta = (\nu^3/\varepsilon)^{1/4}$ is the Kolmogorov scale,

$\varepsilon = 15\nu \left\langle \left| \frac{dv}{dx} \right|^2 \right\rangle$ is the mean energy dissipation rate, ν is the fluid kinematic viscosity and U_o is the mean velocity of the flow [2,3].

In general, one can define the scaling exponents $\zeta(p)$ of the structure functions $S_p(r)$ in the inertial range by the relation (2), or its equivalent scaling expression : $\langle V(L)^p \rangle = \langle L^{\zeta p} \langle \epsilon(L) \rangle^{\zeta p}$. Kolmogorov's 1941 theory (K41)[4], predicts that the statistical properties of $\delta v(r)$ depend only on ε and r, it then follows by dimensional analysis that $\zeta(p) = \frac{p}{3}$, but in recent years many experimental and numerical simulations at very high Re [1], have shown that Kolmogorov scaling is violated in the the inertial range. Therefore, scaling exponents $\zeta(p)$ are a nonlinear function of p [5].

From a practical point of view, the inertial range is defined by the range of scales where the third-order structure function S_3 follows the K41 law:

$$S_3 = -\frac{4}{5}\varepsilon r \quad (3).$$

The larger the Reynolds number, the broader the inertial range but for low to moderate Reynolds numbers accessible to direct numerical resolution, this range is often very narrow.

Extended Self-Similarity (ESS)

The Extended Self-Similarity is a property of the velocity structure functions of homogeneous and isotropic turbulence. It was recognized by Benzi *et al.* [2], that the moments of any order may be plotted as a function of another order, then the scaling is much more pronounced and there seems to be self-similarity for a larger range of scales:

$$S_n \approx S_m(r)^{\zeta(m,n)} \quad (4).$$

where $\zeta(m,n) = \frac{\zeta(n)}{\zeta(m)}$. In the other words, the ratio of two scaling exponents remains constant for a wider range of scales than when taken separately, the reason for this behaviour is still not clear. As was show in [6,7], the ESS scaling comprises not only the inertial range, but reaches as far down as few Kolmogorov scales η.

We have taken advantage of this property to compute scaling exponents $\zeta(m,n)$ with higher accuracy than by spectral methods even at relatively moderate Reynolds numbers. Another important feature of ESS that we exploit in this paper, is that providing information in terms of the relative scaling exponents $\zeta(m,n)$, is universal in geophysical flows, in the sense that they remain valid also in 2D case [8]. But this kind of universality, observed in different flows, disappears if the system is influenced by the presence of strong shear as shown in experiments in wakes behind a cylinder [9] and in boundary layer turbulence [10].

In our study we use $m = 3$ and $\zeta(m) = 1$ derived exactly by Kolmogorov equation, so we determine the scaling of the modulus of any structure function with respect to the modulus of the third order structure function using the following expression :

$$\langle |\delta V(r)|^n \rangle = A_n \quad \langle |\delta V(r)|^3 \rangle^{\zeta(n)} \quad (5).$$

where $\delta V(r) = V(x+r) - V(x)$, and $\zeta(n)$ the scaling exponents of the order n. With the resolution of our data, we were able to study the structure functions up to 6th order.

The experimental apparatus

We study the structure functions and their scaling exponents in decaying non-homogeneous turbulence produced by a grid as a model of geophysical turbulence decaying as we move away from the source of the turbulence. The turbulent velocity fluctuations in an open water flume are measured with an ultrasonic velocimeter SONTEK3-D, (Vx,Vy,Vz). The flume has a test section with a length of 1 m, base cross-section, 0.15 m, and height, 0.3 m. The grid used had a mesh of 0.008 m with a corresponding bar size of 0.002 m and a solidity ratio of 0.34 and was placed 0.06 m downstream of the flow inlet. The mean velocity ranged between 0.04 - 0.3 (m/s). The sampling volume of the sonic velocimeter SONTEK3-D was about 5.10^{-9} m^3 measured at a distance of 0.005 m to 0.008 m from the sensor tips. The sampling frequency was 25Hz. Velocity time series were recorded at a number of control points in the wake of the grid, all of them were centered on the axis of simetry of the tank, at the following downstream distances from the grid : X = 0.105 m, 0.155 m, 0.205 m, 0.255 m, 0.305 m, 0.355 m, 0.405 m and 0.455 m, each time series, with practically constant average mean velocity, in our experiment consisted of around 4000 samples. The velocity time series were transformed from time to spatial domain, this was accomplished by means of Taylor's hypothesis in order to evaluate the structure functions defined in (1). It has to be noted that in presence of large coherent structures Taylor's hypothesis is known to give inaccurate results [11].

Experimental results

The Reynolds number $Re = \frac{u_{mean} L}{\nu}$ was found to vary with the downstream distance from the grid. Its dependence on $\frac{X}{M}$, where X is the distance from the grid and M is the mesh of the grid, is $Re = 630\left(\frac{X}{M}\right)^{-1.27}$ between Re = 11000 and Re = 71000.

The dependence of the structure functions $S_p(r)$ with p = 2,3,4,5,6 on the separation distance r normalized with respect to the Kolmogorov length scale η is shown in Fig.1, for the downstream distance X = 40.5 cm. The structure functions $S_p(r)$ are recovered from the time series by means of the standard Taylor's hypothesis [3], as mentioned above, due to the moderate to low Reynolds numbers in our experiment one can hardly find a range where the spectrum slope remains constant so in principle we can not determine the scaling exponents with the required accuracy. For calculating the scaling exponents for such flows, similar to those encountered in the ocean and the atmosphere, we use Extended self-similarity (ESS), and the structure functions $S_p(r)$ are plotted against S3=⟨|(V(x+r)-V(x))|3⟩. It has also been verified, following Benzi *et al.* [6], that for our data the behaviour S3=⟨|(V(x+r)-V(x))|3⟩ scales in a similar way as: s_3=|⟨(V(x+r)-V(x))^3⟩|. This possible difference due to the use of the absolute value of the velocity signals has centered the reservations on ESS by Streenivasan [1].

We checked the relationship $\frac{S_3}{s_3}$ for different distances from downstream from the grid for our 3D moderate Reynold turbulent flow, as shown in Fig. 2. We show in Fig. 3, the structure functions of order p up to 6th order *versus* $\frac{r}{\eta}$. The data is the same as shown in fig.1 for values $\frac{r}{\eta} > 100$, with $\eta = 0.008$ m, there the scaling exponents could not be determined with the required accuracy but from the representation in Fig. 3 we can observe a much better scaling.

It was found [10] that changes of the limits of the ESS range could influence the values of the scaling exponents, providing the main source of error, it is then necessary to determine the limit of the scaling range, and avoid using the fits outside the inertial subrange, so we define a uniform criteria for the determination of the lower and the upper bounds of the ESS range.We can take the lower bound equal to a certain multiple of the Kolmogorov scale η.