ON THE ESTIMATION OF WIND STRESS FROM MEAN WIND TIME SERIES

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Abstract

The relationship between smoothed wind stress calculated from hourly wind data and the wind stress calculated from smoothed winds is found to be linear, to a high degree (correlation coefficient squared greater than 0.85). A method of estimation of smoothed stress based on linear regression is proposed and successfully tested on 8 years of hourly winds measured at an Adriatic meteorological station.

Key-words: Adriatic Sea, wind

Introduction

Wind blowing over the sea surface is known to be an important forcing agent influencing sea water dynamics. In fact, it is the wind stress, and not wind itself, that acts directly on the sea. This action is manifested at various time scales. Thus, on synoptic time scale (~1-10 days) one finds numerous storm surge and related studies. Subsynoptic (~10 day -1 month) variability also attracted some attention, especially regarding sea-level variations (*e.g.* [1, 2, 3]). The monthly and even longer variability is important for analysis of interannual sea-level fluctuations (*e.g.* [4]). Hence, one often needs sufficiently long time series of averaged wind stress.

The wind stress is commonly calculated through the bulk formula: $\tau = \rho C_D|\mathbf{u}|\bullet\mathbf{u}$

where ρ is air density, C_D is dimensionless drug coefficient and $\mathbf{u}$ is horizontal, averaged (over typically 1 hour period) wind speed vector at some reference height (usually 10 m above the sea surface). It has long been recognized that in order to calculate averaged stress, the bulk formula should be applied to the original hourly, or at least daily, wind values prior to averaging (see [5]). Application of the formula to the already smoothed wind results in significant underestimation of the mean stress. The underestimation is negligible for 1-day averaging and increases by a factor of 2 or 3 for longer periods. Obviously, due to the nonlinear nature of the bulk formula, high-frequency variability of the original wind cannot be ignored. Thus, two wind stresses may be distinguished: Averaged Stress calculated from original Wind (ASW or averaged stress) and the Stress calculated from Averaged Wind (SAW or stress from averaged wind). The first stress, although exact (up to the bulk formula), requires the knowledge of finely sampled time history that is not always available. On the other hand, the second stress is easily obtainable and physically desirable but is wrong as the estimator of the true stress.

It is the purpose of this work to examine more closely, theoretically and practically, the relationship between the two stresses. It is shown that, to a high degree, this relation is linear. Furthermore, it is found that fluctuations, i.e. deviations from the overall mean of the ASW, can be fairly well estimated from the fluctuations of SAW and knowledge of the overall mean and variance of the original wind. On the contrary, the overall mean of the ASW can not be estimated in this way. The above mentioned linearity is found also in a previous paper [6], regarding the calculation of mean bottom stress in the presence of tidal current.

In Section 2 wind is modeled as a two-dimensional, Gaussian process and theoretical results are obtained. The non-Gaussian case is briefly discussed in Section 3 where the results are tested on 8 years of hourly wind data recorded at Split, Croatia. Fairly good agreement is found.

Theory

Let the wind time history be represented by a two-dimensional Gaussian process (u(t), v(t)). For simplicity, as well as uncertainty in the form of C_D, the stress is calculated from bulk formula with constant C_D and ρ. Then, without loss of generality, ρC_D is replaced by 1. Other forms, proposed in literature (see [7] for a review), can be treated as well.

Let us introduce some conventions. The time variable is omitted. The two components of wind as well as of various stresses are denoted by u and v, appropriately indexed. In the same way, mean values are denoted by m and standard deviations by σ. The two basic operations, *i.e.* averaging and calculation of stress, are denoted by subscripts A and S, respectively. For example, u_S is u component of stress calculated from original wind, while $v_{AS} = (v_A)_S$ is v component of stress calculated from averaged wind; m_u is mean value of u component of original wind, while σ_{uA} is standard deviation of averaged u component of original wind. Finally, for any variable u with mean value m, the centered variable is denoted by u', *i.e.* u'=u-m.

Thus, the wind vector (u,v) is supposed to possess bivariate, normal probability density function (PDF).

$$p(u,v) = \frac{1}{2\pi\sigma_u\sigma_v}\exp\left\{\frac{(u-m_u)^2}{\sigma_U^2}+\frac{(v-m_v)^2}{\sigma_v^2}\right\}.$$

Apparently, the variables u and v are uncorrelated, which can always be achieved by appropriate rotation of the coordinate axes (the simplest case of principal component transformtion, see *e.g.* [8]). In the sequel, the discussion is restricted to u component only. The main (computational) step is to examine linear relationship between the wind component u and corresponding stress $u_s = \sqrt{u^2+v^2}\bullet u$. Thus, we may write :

$$u'_s = u_s - m_{uS} = a_u \bullet u' + \varepsilon_u, \tag{1}$$

where the slope a_u is determined by minimizing variance of the error term ε_u. At this point, for simplicity of exposition, it is assumed that the mean values of both components are zero, *i.e.* $m_u = m_v = 0$. For convenience, two cases are distinguished: case (1) if the ratio σ_v^2/σ_u^2 is ≤1, and case (2) otherwise. Correlation coefficient squared between u_S' and u' is calculated, by numerical integration, as a function of σ_v^2/σ_u^2 in case (1) (curve no. 1 on Fig. 1), and as a function of σ_u^2/σ_v^2 in case (2) (curve no. 2 on Fig. 1). It is always remarkably high, *i.e.* greater than 0.8, except in case (2) for $\sigma_u^2 << \sigma_v^2$, when it decreases to 0.61. In the later case, the wind is highly polarized and, probably, may be considered as a scalar. The slope a_u is given by

$$\begin{aligned} a_u &= \sigma_u\, g_1(\sigma_v^2/\sigma_u^2), && \text{in case (1)},\\ a_u &= \sigma_v\, g_2(\sigma_u^2/\sigma_v^2), && \text{in case (2)}, \end{aligned} \tag{2}$$

where the functions g_1 and g_2 are plotted on Fig. 1 (curves no. 3 and 4 respectively). Now, let us consider averaged wind u_A and corresponding SAW u_{AS}. The vector (u_A, v_A) is again bivariate normal. The components u_A and v_A are approximately uncorrelated having original mean values m_u and m_v, but lower variances $\sigma_{uA}^2 < \sigma_u^2$ and $\sigma_{vA}^2 < \sigma_v^2$. Hence, u_A and u_{AS} are again highly correlated, and we may write :

$$u'_{AS} = u_{uA} \bullet u'_A + \delta, \tag{3}$$

where the slope $\mathbf{a}_{uA}$ is obtained from (2) by changing σ_u to σ_{uA} and σ_v to σ_{vA}.

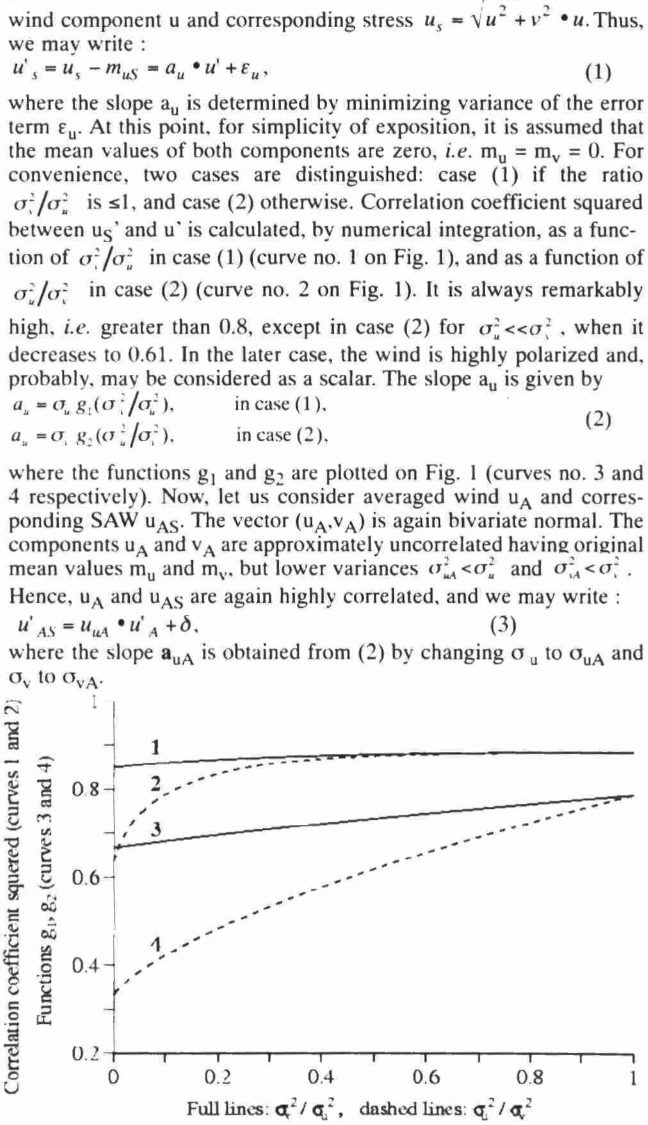

Figure 1. Results based on normal PDF. The correlation coefficient squared between u wind component and corresponding stress u_S (curves 1 and 2). The functions g_1 and g_2 entering formula (2) for the slope (curves 3 and 4).