ON A POTENTIAL VORTICITY IDENTIFICATION OF THE NO-MOTION LEVEL USING ONLY HYDROLOGIC DATA, WITH APPLICATIONS TO THE MEDITERRANEAN SEA

M. Kurgansky [1, 2], and E. Salusti [3]*

[1] Department of Atmospheric and Oceanic Physics, Faculty of Physics and Mathematics, University of Concepcion, Casilla 160-C, Concepcion, Chile - kurgansk@udec.cl
[2] A.M. Obukhov Institute of Atmospheric Physics, Academy of Sciences, Russia.
[3] INFN, Phys. Dept., "La Sapienza" Rome University Piazzale A. Moro, 2
Rome, Italy, * ettore.salusti@roma1.infn.it

Abstract

The theory of potential vorticity for stratified fluids in a rotating system is applied to potential temperature θ and salinity *S*. These 'tracer potential vorticities' are used to obtain an absolute fluid velocity that generalizes earlier formulations. Here we use not the steady absolute velocity but only a vector $\nabla\theta\times\nabla S$ proportional to it. In this way, insight is gained into a purely hydrological identification of the no-motion level. This is then applied to historical data of the Mediterranean Sea with interesting results.

Keywords: potential vorticity, no motion level

A classical problem

It is well known how from the measurements of marine temperature and salinity one can obtain vertical gradients of the current velocity **u** [1], [2]. Recently, oceanographers using the β-spiral theories have greatly improved our theoretical understanding of these problems by noting how in reality hydrologic data allow the identification of the no-motion level [see 3 for a review]. A somewhat simpler relation for steady currents was obtained in [4] that in addition utilizes only potential temperature θ and salinity *S* data set.

Here we restrict ourselves to a simpler results that gives not **u** but only a vector $\nabla\theta\times\nabla S$ proportional to it, to search for surfaces of no absolute motion (NAM in the following) for steady or quasi-steady, inviscid or viscous marine currents. This is then applied to historical data of the Mediterranean Sea with good agreement between our theory and other information.

The absolute velocity field

In a steady adiabatic case, assuming $\nabla\cdot(\rho\mathbf{u})=0$, $\mathbf{u}\cdot\nabla\theta=0$, $\mathbf{u}\cdot\nabla S=0$, calling ρ the water density, the most general representation is

$$\rho\mathbf{u} = \varphi(\theta, S)\,\nabla\theta\times\nabla S, \tag{1}$$

where φ (θ, *S*) is an arbitrary function. We now consider potential vorticities $\Pi_\theta=(\omega_a\cdot\nabla\theta)/\rho$ and $\Pi_S=(\omega_a\cdot\nabla S)/\rho$ with ω_a as the absolute vorticity. Application of the general Ertel's vorticity theorem and also the method [4] gives two formulas

$$\mathbf{u}=\frac{g\rho^{-2}(\partial\rho/\partial\theta)\mathbf{k}\cdot\nabla\theta\times\nabla S}{\nabla\left(\dfrac{\omega_a\cdot\nabla S}{\rho}\right)\cdot\nabla\theta\times\nabla S}(\nabla\theta\times\nabla S)\equiv\frac{g}{\rho^2}\frac{w(\partial\rho/\partial\theta)}{\mathbf{u}\cdot\nabla\Pi_S}(\nabla\theta\times\nabla S) \tag{2}$$

$$\mathbf{u}=-\frac{g\rho^{-2}(\partial\rho/\partial S)\mathbf{k}\cdot\nabla\theta\times\nabla S}{\nabla\left(\dfrac{\omega_a\cdot\nabla\theta}{\rho}\right)\cdot\nabla\theta\times\nabla S}(\nabla\theta\times\nabla S)\equiv-\frac{g}{\rho^2}\frac{w(\partial\rho/\partial S)}{\mathbf{u}\cdot\nabla\Pi_\theta}(\nabla\theta\times\nabla S) \tag{3}$$

where **k** is the unit vertical vector, *g* the gravity and *w* the vertical velocity. According to (1-3) we focus our attention on the vector $\nabla\theta\times\nabla S$. In [4] it is shown that $\nabla\theta\times\nabla S$ is proportional to **u** if the streamlines do not change their shape with time, even if $|\mathbf{u}|$ can be (slightly) time-dependent. This single vector can moreover give sufficient information about the NAM surfaces Σ.

The condition $\nabla\theta\times\nabla S=0$ is satisfied when $\theta=\Psi(S)$ or more generally $\nabla\theta=\Phi(\mathbf{x})\nabla S$, where Ψ and Φ are arbitrary functions. When Σ is essentially inclined to the horizon, as it is typical in the southern Mediterranean Sea, then we may take $\Phi=\Phi(z)$ or better $\Phi=\Phi_1(z)\Phi_2$, as a first guess in order to arrive at $\nabla\theta\times\nabla S=0$ from the above conditions.

No-motion surfaces in the Mediterranean Sea

The Mediterranean Sea has been studied for a particularly long time, since the Phoenician and Greek civilizations. However only in 1961 G. Wüst [5] was able to present a realistic scheme of its general circulation. The data set of θ and *S* we analyze is taken from the Atlas Hydrologique de la Mediterranèe [6], prepared by P. Guibout.

We start with transect 69-70 between the Libyan coast and Sicily (Cape Passero), along 15°40′ E for the winter season. Under the MAW, the LIW core is rather superficial, at 350-400 m depth. There is a sharp zone of strong shears of the current velocity in the upper 200 m, spreading from the Libyan coast till ~35°N. So a reasonable Σ surface can be identified with the *S*=38.60 surface, that is ~250 m deep in the main part of transect between 32°30′-35°00′ N, and then rapidly shoals near the Malta Island.

Transect 65-66 from Cape Dimas, Tunisia, to Malta Island is rather similar to the previous transect: a reasonable Σ surface is at 400-300 m depths, corresponding to an *S* = 38.30 - 38.60 layer while the LIW core is at ~1000 m depth.

The transect 15-16 from Algeria to Toulon, along 6° E, is also interesting. The core of LIW, with *S* = 38.50, is at ~400 m depth. At south one can easily recognize the superficial core of MAW. A reasonable Σ surface is evident only between Algeria, where it is ~170 m deep, and the region at 39°30′-40°00′ N, along the surfaces *S* = 37.70 - 37.90. Near the French coast there is a very strong coastal jet, flowing westward, which doesn't seem to have any no-motion level.

All this look in agreement with the scheme of an eastward superficial MAW surface current at south, flowing opposite to the underlying LIW, and of rather parallel flows of LIW and MAW at the north. However, these are actually debatable points as discussed by C. Millot and other authors in various articles (see [7], for a review).

It is obvious that there are still some serious difficulties regarding the direct implementation of our theoretical results in the study of real data, both from the observational and the computational standpoints. So far, only the Mediterranean Sea, seen as a natural laboratory with its sharp contrasts between different water masses and relatively dense observational network, including historical data sets, can be used to test our ideas, albeit in a preliminary way.

References

1 - Fomin, I.M., 1964. The Dynamic Method in Oceanography. Elsevier Oceanography Series. Elsevier Publ. Comp., Amsterdam-London-NY, 212 p.

2 - Wunsch, C., 1978. The North Atlantic general circulation west of 50∞ West determined by inverse methods. *Rev. Geophys. Space Phys*, 16: 583-620.

3 - McDougall, T., 1995. The influence of ocean mixing on the absolute velocity vector. *J. Phys. Oceanogr.*, 25: 705-725.

4 - Kurgansky, M.V., Budillon G. and Salusti E., 2002. Tracers and potential vorticities in ocean dynamics. *J. Phys. Oceanogr.*, 32: 3562-3577.

5 - Guibout, P., 1987. Atlas Hydrologique de la Mediterranèe. SHOM ed., Ifremer, Paris, 150 p.

6 - Wüst, G., 1961. On the vertical circulation of the Mediterranean Sea. *J. Geophys. Res.*, 66(10): 3261-3271.

7 - Fuda J.L., Millot C., Taupier-Letage I., Send U. and Bocognano J.M., 2000. XBT monitoring of a meridian section across the western Mediterranean Sea. *Deep Sea Res.* I, 47: 2191-2218.