A SIMPLE BAROCLINIC MODEL FOR DIFFUSIVE VORTICES

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Abstract

A simple layered diffusive model describes the temporal evolution of the angular velocity and vertical structure of oceanic vortices. An initial state specifies the core angular velocity of each layer and the density changes between adjacent layers. The temporal evolution of the angular velocity for each layer is explicitly obtained solving the radial diffusive equation, with the diffusion coefficient determined from a stability analysis, and the shape of the interfaces follows from the assumption of gradient flow.

Keywords: oceanic vortices, radial diffusion, inertial stability, layered model.

Transient mesoscalar vortices are a common feature in the Mediterranean. A simple model for vortex evolution consists on radial diffusion of angular velocity $\omega(r,t)$,

$$\frac{\partial \omega}{\partial t} = K \frac{\partial^2 \omega}{\partial r^2}, \qquad (1)$$

where $K(t)$ is a horizontal diffusion coefficient, for an initially rotating cylinder

$$\omega(r, t=0) = \begin{cases} \omega_0, & |r| \le a \\ 0, & |r| > a \end{cases}, \qquad (2)$$

where a and ω_0 are the vortex's initial radius and angular velocity (positive/negative for cyclonic/anticyclonic). The solution, for constant K, is [1]:

$$\omega(r,t) = \frac{\omega_0}{2}\left\{ erf\left[\frac{r+a}{(4Kt)^{1/2}}\right] - erf\left[\frac{r-a}{(4Kt)^{1/2}}\right]\right\} \qquad (3)$$

Figure 1a displays this solution at several times for an initial cyclonic vortex with $a = 25$ km, $|\omega_0| = 3 \times 10^{-5}$ s^{-1} (period 2.5 days), and $K = 90$ m^2 s^{-1}. The size of the vortex slowly increases with time while a central core remains in near solid body rotation. This core remains discernable in time, although after some 30 days its rotation rate decreases.

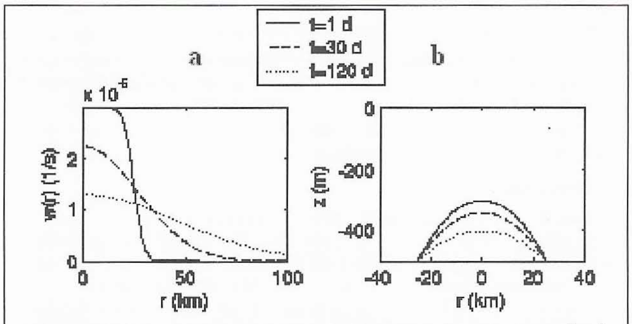

Figure 1. (a) ω at t = 1, 30 and 120 days for K = 90 m^2 s^{-1}, (b) corresponding depths of the upper layer interface h.

The above model may be generalized to a layered ocean (Fig. 2), with a time-dependent $K(t)$ for each layer, provided that $|\partial K/\partial t| << |\omega K|$. The pressure p is hydrostatic and the interfaces' shape may be determined under the assumptions that (a) the deep flow is motionless and (b) the flow in each layer is in gradient balance:

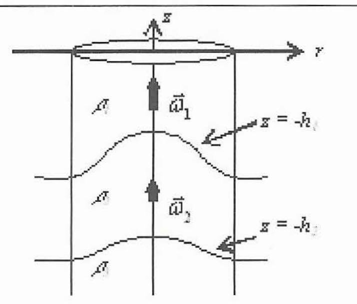

Figure 2. Model schematics.

$$\frac{v_\theta^2}{r} + f\, v_\theta = \frac{1}{\rho}\frac{\partial p}{\partial r}, \qquad (4)$$

where f is the planetary vorticity. For a 1.5 layer model this equation gives the slope of the active only interface,

$$\frac{\partial h}{\partial r} = \frac{\omega(\omega + f) r}{g'}, \qquad (5)$$

where $g' \equiv g(\rho_2 - \rho_1)/\rho_1 \equiv \delta\rho/\rho_1$ and ρ_1, ρ_2 are the upper and lower layer densities. The actual interface depth is obtained through integration from a distant unperturbed radial coordinate, $z = -H$. A first approximation has the vortex in solid body rotation $\omega(r=0,t) \equiv \omega_c$ for $|r| \le a$, the interface $h(r,t)$ becoming a function of ω_c:

$$h = \begin{cases} H + \dfrac{\omega_c(\omega_c + f)(r^2 - a^2)}{2g'}, & |r| \le a \\ H, & |r| > a \end{cases} \qquad (6)$$

Figure 1b illustrates the interface evolution for a cyclonic vortex at 52°N, the sloping interface resembling observations of tilted isopycnals in vortices.

The model requires a knowledge of $K(t)$ for all layers. Within each layer the angular velocity is approximately constant at the vortex core so any diffusion there simply redistributes particles with similar angular velocities. Hence, for each layer the effective diffusion coefficient depends on the stability at the vortex edge, which we assess using the theory of radial stability in barotropic structures [2]. At lowest order the radial velocity is unstable when $\Omega\,\Omega_{sb} < 0$, where $f + 2\omega + r\dfrac{\partial \omega}{\partial r}$ and $\Omega_{sb} \equiv f + 2\omega$.

Figures 3a,b show the radial distribution of $\Omega\Omega_{sb}$ for a cyclonic vortex, at different times, using two different diffusion coefficients. For cyclonic vortices edge stability leads to estimates of $K(t)$: a negative minimum at the vortex edge is interpreted as a too small coefficient (Figures 3a,b). For anticyclonic vortices the eddy's edge is rather stable leading to a small effective diffusion [1].

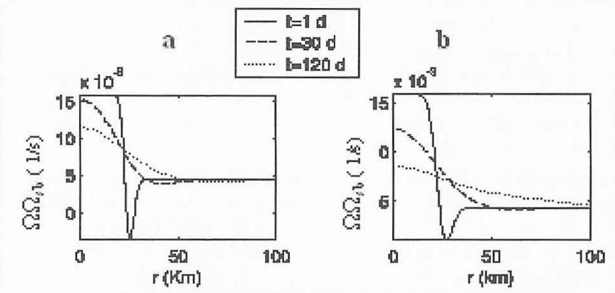

Figure 3. $\Omega\Omega_{sb}$ radial distribution at t = 1, 30, and 120 days for (a) K = 30 m^2 s^{-1} and (b) K = 90 m^2 s^{-1}.

References

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