ECOLOGICAL AND MATHEMATICAL ASPECTS OF A PLANKTON COMMUNITIES MODEL

Jean-Marc Ginoux *, Bruno Rossetto, Jean-Louis Jamet
Université de Toulon et du Var, Equipes ISO-EBMA, Laboratoire PROTEE, LA GARDE cedex, France
* ginoux@univ-tln.fr , rossetto@univ-tln.fr, jamet@univ-tln.fr

Abstract
In this article we studied an ecological model transcribing the behavior of a trophic chain with three levels of interaction. The values of the parameters of the model lead us to regard it as a slow fast autonomous dynamic system. We could establish the equation of the slow surface on which the attractor is leaned, thus connecting the three variables in the slow field in the form of a state equation.

Key words : Plankton modeling, Slow-Fast Dynamics

The ecological point of view

Since 1995, seasonal variations of the plankton community (phytoplankton and zooplankton) in Toulon Bay (Var, France) have been studied [1, 2]. Plankton communities differently affected by anthropogenic inputs of Toulon area have been studied regularly in two sites (Little Bay, polluted, and Large Bay, non polluted).

The average of nitrate and orthophosphate concentrations and the abundance and diversity of plankton communities during annual cycles in these bays exhibit inherent features as the pattern shape of limit cycles as well as the apparent period of intrinsic evolution of phytoplankton and zooplankton separately considered.

So, we have searched a model able to transcribe the observed behaviour of plankton communities and to fit with our data.

Among the different autonomous predator-prey models taken under consideration in the literature [3-10], we searched the most *simple and consistent* model with functional responses for growing, predation and mortality susceptible to lead to a limit cycle and to fit with our data. To this aim, we were interested by the Rosenzweig-Mac Arthur model [11].

Modelling

The Rosenzweig-Mac Arthur model [11] for a three trophic level interaction is involving nutrients (nitrates and orthophosphates, N), planktonic algae (phytoplankton, P) and herbivorous zooplankton (Z). It's the so-called NPZ model. It is composed of limited functional responses : a logistic prey (N), a Holling type II predator (P), and a Holling type II top-predator (Z). Predator's per-capita predation rate has the Holling type II form [10]. All parameters used are chosen in a biological range.

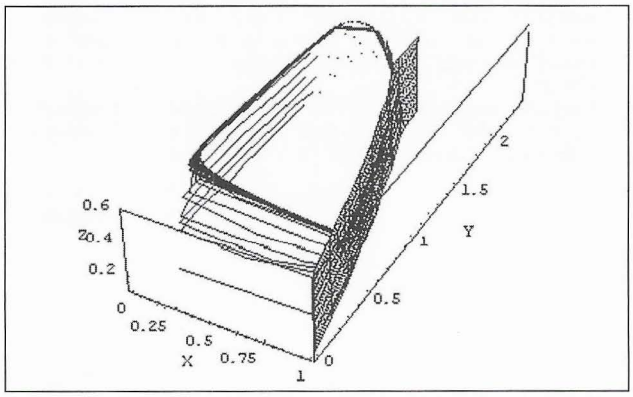
Note that the Rosenzweig-Mac Arthur model was developed from the previous works of Volterra [4] and Lotka [5]. The addition of limited functional responses made it possible to lead to the model of Rosenzweig-Mac Arthur. Nevertheless, if only one limitation term is removed, there is no more limit cycle.

Moreover, as the turn-over of each variable is in a ratio of ten, we have been lead to focus our interest on slow-fast autonomous dynamic systems [12, 13, 14].

Mathematical aspects

Under certain conditions, the dimensionless system is singularly perturbated with three times scales. The rates of change for the prey, the predator and top-predator range from fast to intermediate to slow, respectively [12, 13]. We give the equation of a slow manifold on which the attractor lies. Since, a state equation relying the three variables is established.

Some simplications could be brought to the model of Rosenzweig-Mac Arthur in order to make a study in two dimensions of it.



References

1 - Jamet J.-L., Ferec-Corbel A.-S., 1996, Seasonal variations of the zooplankton community in a littoral marine ecosystem : Toulon Bay (Var, France). *Mar. Life,* 6: 15-20.
2 - Jamet J.-L., Bogé G., Richard S., Geneys C., Jamet D., 2001. The zooplankton community in bays of Toulon area (northwest Mediterranean Sea, France). *Hydrobiologia*, 457: 155-165.
3 - Verhulst P.F., 1838. Notice sur la loi que suit la population dans son accroissement. *Corresp. Math. Phys.*, X: 113-121.
4 - Volterra V., 1926. Variazioni e fluttuazioni del numero d'individui in specie animali conviventi. *Mem. Acad. Lincei*, III: 31-113.
5 - Lotka A.J., 1925. Elements of Physical Biology. Williams and Wilkins, Baltimore.
6 - Gompertz B., 1825. *Phil. Trans.*, 115: 513.
7 - Gause G.S., 1934. The struggle for existence. Williams and Wilkins, Baltimore.
8 - Watt K.E.F., 1960. *Can. Entomol.*, 92: 674.
9 - Ivlev V.S., 1961. Experimental Ecology of the Feeding Fishes. Yale Univ. Press, New Haven, Conn.
10 - Holling C.S., 1959. Some characteristics of simple types of predation and parasitism. *Can. Entomol.*, 91: 385-398.
11 - Rosenzweig M.L., and Mac Arthur R.H., 1963. Graphical representation and stability conditions of predator-prey interactions. *Am. Nat.*, 97: 209-223.
12 - Deng B., 2001. Food chain chaos due to junction-fold point. *Am. Inst. Phys.*, 11: 514-525.
13 - Ramdani S., Rossetto B., Chua L., and Lozi R., 2000. Slow manifold of some chaotic systems - laser systems applications. *Int. J. Bifurc. Chaos*, 10: 2729-2744.
14 - Rinaldi S., and Muratori S., 1992. Slow-fast limit cycles in predator-prey models. *Ecol. Model.*, 61: 287-308.