REMINERALIZATION PROCESS IN A NUTRIENT-PHYTO-ZOOPLANKTON-BACTERIA MODEL

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Abstract

Seasonal variations of plankton communities in the Toulon Bay (Var, France) have been studied since 1995 [1, 2] and samplings have been carried out regularly. The abundance of phytoplankton (P), zooplankton (Z) and nutrients concentration (N), *i.e.* average concentrations of nitrates and orthophosphates have lead to a three-trophic model [3]. Since 2005, bacteria average concentrations (B) have been also evaluated to take into account their influence on plankton communities in the remineralization process. Thus, the aim of this article is to propose the most simple and consistent Nutrient-Phyto-Zooplankton-Bacteria model transcribing this process. Then, regarded as a dynamical system its mathematical features are highlighted.
Keywords: Population Dynamics, Nutrients, Phytoplankton, Zooplankton , Bacteria

The biological point of view

The classical three-trophic food chain [6] can be modelled by interactions of predator-prey type which consist of a top-predator (Z) which can graze on phytoplankton (P) which on its turn can predate on nutrients (N). But, bacteria can break down into utilizable nutrient, a process known as remineralisation [4]. Then, zooplankton can graze either upon phytoplankton or bacteria or both. Many models have been already developed in order to transcribe such behaviour [4] but the functional responses that involve make their mathematical study and biological interpretation difficult. So, starting from such seminal works the most simple and consistent NPZB model is proposed.

Modelling

Let's consider the classical Volterra predator-prey model [5, p.88] for a three trophic level interaction involving nutrients (N), phytoplankton (P) and zooplankton (Z) and let's include bacteria (B). This NPZB model does not include any functional response with limitation such as Holling type II or type III [4] since it can be considered that bacteria are playing the role of limiting factor in the evolution of this system. By considering that the remineralisation process can be modelled by an interaction of predator-prey type two terms are added to the nutrients growth: PB and ZB which correspond to the transformation of phyto and zooplankton losses into nutrients by the bacteria. Of course such terms represent an uptake for the bacteria. The most simple and consistent NPZB model is thus built with a parameter set chosen in a biological range.

Mathematical aspects

The behaviour of such dynamical system transcribes the evolution of each of its components. And since for certain parameter values this singularly perturbed system has three times scales the rates of change for the prey (N), the predator (P), the top-predator (Z) and the bacteria (B) range from fast to intermediate to slow, respectively [7]. The slow manifold analytical equation on which the trajectory lies (limit cycle or chaotic attractor) is provided according to a new method developed in [8]. Then, a state analytical equation involving the four variables is established.

References

1 - Jamet J.-L. & Ferec-Corbel A.-S., 1996. Seasonal variations of the zooplankton community in a littoral marine ecosystem : Toulon Bay (Var, France), *Mar. Life*, 6(1-2), 15-20.

2 - Jamet J.-L., Bogé G., Richard S., Geneys C. & Jamet D., 2001. The zooplankton community in bays of Toulon area (northwest Mediterranean Sea, France), *Hydrobiologia*, 457,155-165.

3 - Ginoux J.-M., Rossetto B. & Jamet J.-L., 2005. Chaos in a three dimensional Volterra-Gause model of predator-prey type, Int. *J. Bif. & Chaos*, 5(15), 1689-1708.

4 - Edwards, A., 2001. Adding detritus to a nutrient-phytoplankton-zooplankton model: a dynamical-systems approach, *J. Plankton Res.* 23 (4), 389–413.

5 - Volterra V., 1931. Leçons sur la Théorie Mathématique de la Lutte pour la Vie, Gauthier-Villars, Paris.

6 - Holling C.S., 1959, Some characteristics of simple types of predation and parasitism, *Can. Entomologist*, 91, 385-398.

7 - Deng B., 2001, Food chain chaos due to junction-fold point, *Am. Inst. of Physics*, 11 (3), 514-525.

8 - Ginoux J.-M., 2009. Differential Geometry Applied to Dynamical Systems, World Scientific Series on Nonlinear Science, Vol. 66, series A.