# EXAMINING LINEAR TRENDS: EXAMPLES OF SEA LEVELS IN THE MEDITERANEAN

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## Abstract

Long term linear trends for mean regional annual sea levels in Mediterranean are assessed with four models assuming the existence of linear trend vs. no trend and correlated vs. non-correlated residuals. Bayes methodology is applied. Positive autocorrelation is found in all regions which considerably increases the uncertainties of estimated trends. Model comparison strongly indicates the existence of (linear) trends in all regions except in the Aegean sea, where the data slightly prefer the no-trend model.

*Keywords: Sea level, Mediterranean Sea*

The assessment of trends is an important task in the analysis of long term variability of geophysical time series. Due to their simplicity, linear trends are often employed, at least as a starting model. One is interested in the estimate of the slope of the trend line as well as the uncertainty of this estimate. The former is usually obtained by least squares technique (although robust alternative exists [1]) while the latter is obtained from the assumed statistical model of residuals. Most often it is assumed, at least implicitly, that the residuals are equally (normally) distributed and mutually independent. Both assumptions could be questioned, but it is the assumption of independence that is usually violated in geophysical time series. This does not affect the trends themselves, as calculated by least squares, but usually has a strong impact on the estimated uncertainties. For this reason it is necessary to model the autocorrelation of residuals. The problem is sometimes approached within the so called generalized least squares, but the procedure is heuristic and involved. Another possibility is to apply the Bayes methodology, where various assumptions about the residuals can be implemented with relative easy [e.g. 2]. In Bayes approach one does not estimate the 'significance' of trend or any other particular model and/or hypothesis. Instead, it is possible to compare how much the two or more models are favored by the data at hand.

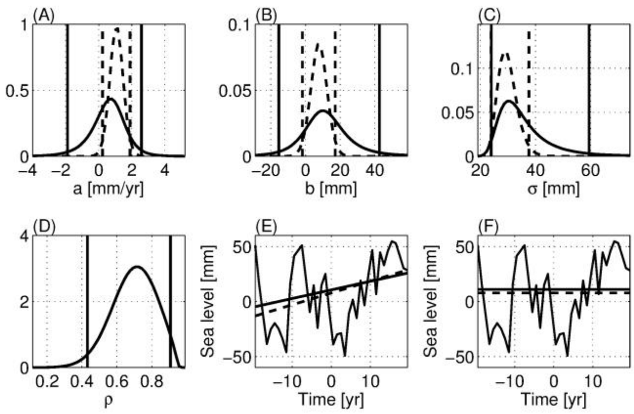


Fig. 1. Posteriors of parameters for models M1(full) and M2 (dashed) (A-D) and corresponding linear trends (E). Constant fits within models M3 (full) and M4 (dashed) (F), all for region R4.

The data analyzed in the present study comprise five time series of mean annual values of sea level for five regions related to the Mediterranean area: Atlantic ocean in the vicinity of Gibraltar strait (R1), West Mediterranean (R2), Adriatic Sea (R3), Aegean Sea (R4) and Black Sea (R5). These series are obtained by averaging the tide gauge data from stations belonging to the particular region. Among all stations available at [3], we retained those with correlation coefficient between neighboring stations being sufficiently high, depending on the region. Prior to the averaging, zero level at each station was adjusted by subtracting the long term mean value calculated over the longest interval that contains the data from all stations. The obtained time series for regions R1- R3 and R5 span the 1927-2008 interval. For R4 the interval is 1969-2008.

We consider a model of linear trend with correlated residuals:

$$y_t = a\,t + b + \epsilon_t$$

where $y_t$ (mm) is sea level, $t$ is time (years, zero corresponds to the middle of the time series), $a$ is slope and $b$ is intercept. The residual $\epsilon_t$ is modeled as Gaussian AR(1) process with zero mean, unknown standard deviation $\sigma$ and unknown lag-one auto-correlation $\rho$. From this model, denoted here as M1, three additional models are obtained by assuming $\rho = 0$ (M2), $a = 0$ (M3) and $a = \rho = 0$ (M4). Thus, M2 is classical linear model with uncorrelated residuals, while M3 and M4 are constant models with correlated and non correlated residuals, respectively. Uniform priors are used in all calculations.

Figure 1 (A-D) depicts the posterior distributions of parameters for models M1 (full) and M2 (dashed line), together with corresponding 95%-credible intervals. As expected, these intervals are considerably wider if one allows the residuals to be autocorrelated. The posterior for $\rho$ (Figure 1 (D)) suggests that likely the lag-one correlation coefficient falls between 0.6 and 0.8 and almost certainly is greater than 0.4. For M2, the modes of posteriors for $a$, $b$ and $\sigma$ are almost equal to corresponding values obtained by classical least squares. The same is valid for credible intervals and classical confidence intervals (assuming the non correlated residuals). Results for other regions and also for (no-trend) models M3 and M4 are similar. Next, the evidences for all models and all regions were calculated and scaled so to set the value at M1 to 100 (Table 1). The M1 model is always preferred (by the data) over the M2 in accordance with the estimated auto-correlation. The M1 is preferred over the M3, except in R4, where the data mildly prefer M3 (3.6:1). Also, in R4, the M2 is slightly preferred over the M4 (2.4:1), so if autocorrelation is not taken into account, the linear trend will be indicated. In classical approach, assuming no auto-correlation, the Kendal's tau p-values [1] (last row in Table 1) would imply significant trends in all regions.

Tab. 1. Evidences (scaled) and p-values for all models and regions (see text).

| Model\Region | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| M1 | 100 | 100 | 100 | 100 | 100 |
| M2 | 5.4 | 0.64 | 8.4 | 0.012 | 9.6 |
| M3 | 0.0049 | 0.16 | 1.34 | 361 | 0.12 |
| M4 | 3e-18 | 1e-13 | 3e-06 | 0.0050 | 3e-09 |
| p-value | 2e-16 | 2e-12 | 1e-08 | 0.013 | 1e-10 |

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## References

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